The Economics of Invention Incentives: Patents, Prizes, and Research Contracts

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Though public intervention in the market for research is virtually universal, economists have paid surprisingly little attention to the choice of the form of research incentive in a given market structure. Many studies concentrate on patents, but any assumption of their superiority over other incentives has been founded on intuition rather than on formal analysis.

In this paper I analyze the choice between three of the most common alternative means of public intervention in the research market, namely, patents, prizes, and direct contracting for research services. I show why, and under what conditions, any one of the three may be preferred by a social welfare-maximizing administrator in a competitive economy, using a model that, for the first time, pays explicit attention to differences in the informational roles of each of these alternatives.

In the extensive literature on the economics of patents (see Arnold Plant, 1934; Fritz Machlup, 1958; Charles Taylor and Z. A. Silberston, 1973; Morton Kamien and Nancy Schwartz, 1975; and F. M. Scherer, 1977, for valuable surveys), formal analysis weighs the benefits of patents as a solution to the market failure associated with the inappropriability of knowledge against the welfare cost due to the restriction on the use of the knowledge generated, and this tradeoff is optimized by patent life adjustment in William Nordhaus (1969). Scant analytical attention is paid to alternative incentive mechanisms. (An exception is Ben Yu, 1981, who considers the role of prior contracting for inventions.)

But as many writers (for example, Dan Usher, 1964; Yoram Barzel, 1968; Joseph Stiglitz, 1969; Carole Kitti, 1973; Glenn Loury, 1979; Partha Dasgupta and Stiglitz, 1980a) have pointed out in various contexts, the incentive offered by an unlimited patent to competitive researchers may be excessive, due to the "common pool problem" discussed further in Section I below. If the patent administrator and researchers share the same information, as implicitly assumed in previous models, then the patent life limitation can be adjusted to provide the optimal patent incentive, given the common pool problem. But in all such models, patents would not be chosen in a fully optimized fiscal system. Researchers and the administrator are assumed to have identical information about the shadow price of potential inventions; a patent is just a means of turning this shadow price into a monetary reward. But monetary compensation can instead be offered directly to researchers by the state. Assuming that patent revenues incur a higher deadweight loss than an equivalent amount of public funds financed by less distortionary means (for example, a minimally efficient tax system), appropriate prizes or government contracts are socially preferable to patents with optimal lives.

If the patent is ever to be the optimal incentive mechanism for research, it must possess advantages not captured in existing models. Informal discussions of patents emphasize their informational role. To include the latter as a justification for decentralized invention incentives, I incorporate an *ex ante* imbalance of information about costs and benefits of research in the model presented in Section II. But this alone is not quite enough. It is further necessary to specify that the terms of the award must be fixed before

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this informational imbalance is resolved, if it can ever be economically resolved. Several arguments for this assumption which is maintained throughout are presented in Section V; if it does not hold, any rationale presented here for choosing patents over other incentives with lower excess burden collapses.

The informational asymmetry of the model in Section II does make a patent system a relevant choice, in contrast to previous models. But it also transforms the common pool problem, previously viewed as correctable, into such a serious drawback for prizes and patents that contracts may be a superior alternative. Hence any of the three alternatives considered may be the best choice given the three allocative difficulties recognized in the model—lack of appropriability of knowledge, deadweight loss of the patent, and the common pool problem. The relationship between the parameters of the model and the choice of incentive mechanism is analyzed in Section III and illustrated for a simple case in Section IV. Some generalizations of the results are discussed in Section V, and conclusions follow in Section VI.

I. Resource Allocation in a Simple Competitive Invention Model where All Agents have Identical Information

I consider the search for a specific process invention which, by lowering the cost of production of some competitively produced commodity, could produce a social benefit of value B. The invention is discrete (a "pot of intellectual gold") and its discovery imparts no knowledge that could be useful in the development of other technologies.[1] A public administrative authority, responsible for research activity, seeks to maximize a risk-neutral individualistic social welfare function through intervention in the market for research. At time 0, the administrator announces the type and terms of the incentive offered to researchers, and researchers make their allocative decisions. All research projects take a fixed time to complete, and finish at time I, when the results of the research are revealed, and the invention, if achieved, is utilized and yields its social benefit. Discounting of costs and benefits is implicit; all prices are expressed as time I present values.

Research activity is carried out by a large number of small independent profit-maximizing, price-taking research firms with identical technologies. One of the factors which they employ, "research skills," measured in efficiency units, has a nondecreasing marginal cost to the research industry, and all the other factors used by researchers are in perfectly elastic supply.[2] For any firm i the total cost of producing μ_i units of research activity is $c(\mu_i, m)$, where m is the aggregate research produced by all firms. Because of the indivisibility of the services of the owner-manager, marginal cost $\partial c/\partial\mu_i \equiv c_1(\mu_i, m)$ first decreases then increases with μ_i; minimum average cost occurs at output μ_i^*, independent of m. For simplicity it is assumed μ_i^* equals unity, so with free entry each firm in the research industry performs just one project and the number of firms equals the number of projects m.[3]

[1]By considering a process invention, I ignore effects of different incentives on product variety which could be important for product inventions, as one referee has noted. Further, the assumption of a single goal greatly simplifies the analysis by ruling out two phenomena related to the disclosure inherent in the patent incentive. The first is that competitors can often "invent around" a patent, lowering its value to the inventor (Machlup, p. 50). This possibility in turn tends to broaden the scope of claims made by inventors, and to encourage defensive "sleeping patents" (Richard Gilbert and David Newbery, 1982). It can also make the alternative of attempting to maintain secrecy more attractive. The second is that inventions often provide some of the information needed to develop other, possibly quite different, socially beneficial inventions, which may not affect the private value of the first discovery. The additional social return to the invention will not be reflected in the value of the patent.

[2]Research skills may be owned by the researcher firms themselves. The supply situation is analogous to that of competitive farmers who own or rent land which has an upward-sloping supply curve to the industry as a whole.

[3]It is assumed that μ_i^* is sufficiently small to justify treatment of aggregate research m as if it were a continuous variable. The equilibrium result, that all firms operate at optimal efficiency, is derived in Dasgupta and Stiglitz (1980a, pp. 17–18).

For the industry as a whole, the cost of m units of research activity is $C(m)\equiv\Sigma c(\mu_i)$ and the industry research activity supply curve is $C'(m)>0$ with $C''(m)\geqslant 0$. In the search for the invention, I assume that if duplication of research tasks occurs, it must be unforeseeable *ex ante* to both researchers and the administrator. (Thus a possible advantage of centrally controlled research contracts in reducing duplication is ignored.) The probability that the invention will be found (i.e., that at least one research project is successful) is $P(m)$, with $P'(m)>0$ and $P''(m)<0$.

A completely decentralized market will not in general yield the optimal level of research in this model. If secrecy is ruled out,[4] the invention, being available at no cost to all users, will have no commercial value. Therefore government intervention is needed to provide private research incentives. The socially optimal allocation of research effort depends upon the information available to the administrator and researchers. In this section it is assumed that the government and private researchers have identical information. They know the cost and probability-of-success functions, and have rational expectations about m prior to the conduct of research.

Then assuming lump sum taxation is possible (or the marginal welfare cost of general revenue is negligible) and there are no market distortions in other sectors, the socially optimal level of research activity m_0 is the level at which expected marginal social benefits $BP'(m)$ equal marginal social cost $C'(m)$. This can be achieved in several ways. The administrator can announce that he will buy m_0 units of research at a price determined by competitive bids. Or, if $C''(m)>0$, he can offer to pay the *ex ante* expected marginal product, $B\cdot P'(m_0)$ for any units of research offered. Then (assuming no alternative demand for research) in equilibrium

$$(1)\quad C'(m_0)=\mu_i^{-1}c(\mu_i,m_0)$$
$$=c_1(\mu_i,m_0)=\partial[\mu_i\cdot B\cdot P'(m_0)]/\partial\mu_i$$
$$=B\cdot P'(m_0),$$

and the optimal level of research output m_0 is achieved with $\mu_i=1$ for any firm i conducting research. Note that the competitive researcher ignores the marginal effect of his own research activity μ_i on $P'(m)$, consistent with the familiar price-taking behavior of an atomistic competitor.

Alternatively, the administrator might consider rewarding successful projects. This can be done by offering patent protection for a successful inventor, or by offering a prize.

Absent any deadweight loss, it might seem that an optimal patent policy would protect the full value of the invention, (see, for example, Nordhaus), and that a prize of equivalent present value would also be optimal. However, in a competitive model either incentive will create a welfare loss in the invention market if an award is made whenever success is achieved and, in the case of multiple successes, is shared equally by the successful projects or allocated randomly to one of them. Since each of the m projects is considered equally likely to discover the invention, the researchers, having rational expectations, consider each project to be one "ticket" in the "invention lottery," the expected return on each being $m^{-1}B\cdot P'(m)$. Producer i equates this expected return to his marginal research cost $c_1(\mu_i,m)$, so that

$$(2)\quad C'(m)=c_1(\mu_1,m)$$
$$=\partial(\mu_i\cdot m^{-1}B\cdot P(m))/\partial\mu_i$$
$$=m^{-1}B\cdot P(m).$$

Since $P''(m)$ is negative by assumption, $m^{-1}P(m)$ is strictly greater than $P'(m)$, so a comparison of equation (2) with (1) shows that if the patent or prize offers the full social value of the invention, excessive resources are allocated to research. In equi-

[4]Secrecy is technically feasible only for the limited class of inventions in which exploitation does not imply revelation of the secret. It should be clear from the discussion below that secrecy is for several reasons socially undesirable in the full information model of this section. See Taylor and Silberston, ch. 9; Edmund Kitch (1977); my 1981 paper; and Steven Cheung (1982) for further discussion of the public and private merits of secrecy as a means of protecting inventions.

librium, the only net social gains expected from research are the rents accruing to the research skills employed by research firms; if these skills are in perfectly elastic supply, the entire research effort produces no net expected benefit at all. The problem is equivalent to that noted by H. Scott Gordon (1954) with respect to competitive fishing in an open access fishery;[5] it is, borrowing Dasgupta and Stiglitz's terminology, denoted the "common pool problem" below.

The common pool problem can be solved by reducing the value of the patent or prize award to a fraction Y of the social value of the discovery, where Y equals the ratio of expected marginal to expected average benefit of research activity. In a multiperiod context, the reduction in patent value would be equivalent to the existing patent life limitation in the United States, except that the length of life would not be fixed at seventeen years, but would be adjusted *ex ante* so that the expected present value of the award is reduced by a fraction $(1-Y)$. Patent life reduction as a solution to the common pool problem is presented in Kitti.

Assuming intertemporally separable patent demands, the analytical equivalent of this patent life limitation in my one-period model is achieved by randomly allowing the award of a patent, given success, only with probability Y which is a function of the parameters of the problem. (A tax on the patent has a greater expected welfare loss.) Except where otherwise indicated, in this paper "patents" denote incentives optimally adjusted in this way.

In this starkly simple static model, there is another way of overcoming the common pool problem with patents and prizes, which, though of dubious practicality, will provide us with a useful yardstick in following sections of this paper. It involves a change in the rules of awarding the patent right or prize. The misallocation illustrated above arises because the expected private benefit from a unit of research exceeds its marginal social benefit. When does a unit of research have a positive marginal social benefit *ex post*? Only when it results in success *and* all other units fail. If more than one researcher discovers the single discrete invention considered here, then the *marginal* contribution of *each* is zero. Hence, the requisite rule change is that the prize or patent right be awarded only in the case of a single, unduplicated success in achieving the research goal at time I.[6] Since $P(m)$ is defined as the probability that *at least one* research project succeeds, each researcher, having rational expectations as to m, will perceive the incentive to be $B \cdot P'(m_0)$, which is exactly the same as the optimal public payment for research described above.

In this model the principle is implemented by mandating that to qualify for an award, all researchers must submit their results in confidence to the planner at the end of the invention period. Competition should rule out any communication between successful applicants which would reveal multiple successes. For want of a better term, we call awards under this rule "marginal patents" and "marginal prizes." In a more realistic dynamic intertemporal context, this method of making awards presents serious administrative challenges. How is parallel invention to be distinguished from copying? Over what time interval can the occurrence of similar inventions be labeled "simultaneous" and no award allowed? How soon would the invention have been found by another researcher? Such difficulties would ordinarily preclude real world implementation of marginal pat-

[5] The application of Gordon's insight to research was noted by Usher. In an intertemporal context the benefits are dissipated in the "race to be first" (Barzel). Others who have recognized the problem in various guises include Kitti, Loury, and Dasgupta and Stiglitz (1980a). Recently Kitch and Yu have assessed patent incentives from an institutional and contractual viewpoint. In the form of an employment contract, prior contracting is much like my "contracts" option (for example, Yu, p. 221). If prior contracts are for patents, moral hazard problems regarding self-identification and motivation of researchers could be severe under informational asymmetries.

[6] That is, research is rewarded only if it makes a marginal difference, *ex post*. The method is in this sense analogous to the assessment of the "Clarke tax" and related mechanisms for obtaining honest revelation of preferences for public goods (Edward Clarke, 1971). I am indebted to Susan Rose-Ackerman for pointing out this analogy.

ents and prizes unless, for some exogenous reason, competition must end and implementation begin at a fixed point in time. However, marginal patents and prizes are useful analytically because, unlike the alternative of reducing the reward, they yield conventional welfare results regarding patents even when informational asymmetry is introduced. They serve as yardsticks which help show, in Sections III and IV below, the extent to which recognition of the common pool problem reduces the value of the more realistic decentralized alternatives under various parameterizations of the model to be introduced in Section II.

Though the above alternative eliminates the common pool problem for patents in the model used in this section, it has no relevance for identification of the best invention incentive system because its associated deadweight loss renders it inferior to direct payments by the public in the form of prizes or contracts.[7] Justification for using patents must be sought in a different model.

The rich informal literature on the patent system emphasizes the importance of the patent as a decentralized decision-making device, implying that information and its distribution are major elements in the rationale for the patent instrument. This rationale is explored in the model outlined below, in which the research administrator possesses information different from that available to private sector researchers. The model is used to evaluate the three alternatives which have widely used, real world counterparts: patents, prizes, and research contracts.[8]

II. The Invention Model with Asymmetric Information

This model is an extension of that above, modified to make information about costs and rewards asymmetric. Total industry research cost is now subject to a random multiplicative disturbance θ, with zero mean and finite variance σ_θ^2. This cost function is denoted $K(m, \theta)$, where $K(m, \theta) = (1 + \theta)C(m)$, with $K_1 \equiv \partial K/\partial m > 0$, $K_{11} \equiv \partial^2 K/\partial m^2 \geqslant 0$. The social payoff if research is successful, B, is subject to an independent random disturbance ζ with zero mean and finite variance σ_ζ^2. Consistent with Section I, the probability that one or more researchers find the single objective is $P(m)$, with $P'(m) < 0$, $P''(m) > 0$.

As discussed above, an imbalance of information between the public research authority or administrator and the innovating agents is essential if a patent system is to be a candidate for the best incentive choice in a competitive model. Reasons for such an imbalance have been advanced by Kenneth Arrow (1962), Martin Weitzman (1974), Richard Nelson (1982), and many others. The assumption that information is costly to acquire, and the implication that it might not always be optimal (or even possible) for the administrator to find out everything relevant to his decision that the private researchers know, seem especially appropriate in the case of inventive activity.

The informational asymmetry is represented as follows. At time 0, when decisions are made on the level of incentive and on the allocation of resources, the researchers know ζ and θ which affect benefits and costs, re-

[7]Stiglitz presents a similar argument. Ward Bowman (1973, ch. 5) points out that price discrimination by the patent holder would reduce the excess burden, but he does not claim that perfect price discrimination is feasible. Assuming all price discrimination is ruled out, the welfare cost caused by a patent on a cost-reducing invention can be described as follows. For simplicity, I assume that the invention is of the run-of-the-mill type (Scherer's 1965, and Nordhaus' terminology) in which the absolute values of the cost reduction and the demand elasticity for the product are not so large that revenue-maximizing output exceeds the preinvention competitive supply, Q_0. (For a clear discussion, see Nordhaus, pp. 70–73, or Dasgupta and Stiglitz, 1980a). The invention lowers marginal (and average) cost from MC_0 to MC_1. A profit-maximizing patent holder maintains output at Q_0 rather than at its new competitive level Q_1. Patent revenue and the approximate welfare cost of the patent, are $Q_0(MC_0 - MC_1)$ and $\frac{1}{2}(Q_0 - Q_1)(MC_0 - MC_1)$, respectively.

[8]Obviously these are not the only means which can conceivably be used to foster research. I have, for example, ruled out complicated combinations of patent lives and subsidies or taxes mentioned in the literature (Kitti, p. 31; Nordhaus, p. 89), and secrecy (see fn. 4 above). An auction system is presented by Barzel and used by Kitti, but "only as a standard of comparison for the current patent system. It is not being promoted as a feasible alternative" (p. 68).

spectively, but the administrator knows only the distribution of ζ and θ. For example, ζ might represent a marketability factor that is known only to private firms familiar with marketing, while θ might reflect industry-specific research experience. Processes which could directly transfer information about these random variables between researchers and the administrator are assumed to be too costly to be feasible.

III. A Comparative Evaluation of Patents, Prizes, and Contracts

The evaluation criterion is the expected net differential social benefit relative to a contract by the administrator for m_0 units of research, the optimal contractual allocation, given that the administrator cannot observe the realizations of ζ and θ. Two types of award are considered initially.

[1] *Patent*. The patent award is given to one of the successful discoverers at time I, (or shared by all successful discoverers, who exploit it as a monopoly), so that the competitive individual inventor responds to the average expected benefit. The welfare loss caused by an *unlimited* patent, which is incurred only when a patent is awarded, is defined as L, where the magnitude of L/B is assumed at most to be of the same order as σ_ζ^2. The common pool problem discussed above is alleviated by allowing the patent award, given success, only with some probability Y, less than unity. As discussed above, this procedure is the atemporal equivalent of limiting patent life; it maintains an equivalent relation between the private value of the patent and its deadweight loss.

In the model of Section I, the optimal allocation m_0 could be achieved by offering to purchase research inputs at the price,

$$B\cdot P'(m_0)=Y_0BP(m_0)/m_0, \tag{3}$$

where Y_0 is equal to $h(P)$, the elasticity of total probability that at least one research project succeeds, evaluated at m_0,

$$Y_0=h(P)\equiv m_0P'(m_0)/P(m_0). \tag{4}$$

In the case of a patent awarded in the current model with asymmetric information, the value of Y, which is optimized by the administrator *ex ante*, is a function of the excess burden L, as well as the other parameters of the problem.

[2] *Prize*. If a prize is chosen, it is awarded whenever success is achieved. If several projects are successful, it is shared by all or a subset chosen at random. The prize value is optimally reduced *ex ante* to counter the common pool problem. I assume that the revenue needed to finance the prize can be raised with negligible excess burden.

The method of evaluation of patents and prizes proceeds as a generalization and extension of the approach used by Weitzman (1974) to study the choice of prices vs. quantities in controlling pollution.[9] First consider the simple example in Figure 1, in which the benefits are nonstochastic and equal to B. The marginal cost of research activity, $C'(m)$, is subject to a multiplicative shifter $(1+\tilde{\theta})$, where $\tilde{\theta}$ equals $+\hat{\theta}$ or $-\hat{\theta}$, each with a probability of one-half. Probability of one or more successes is a nonlinear function $P(m)$, and $BP'(m)$ is the marginal expected return function.

Under a prize option, the administrator optimizes the fraction Y of the social benefits incorporated in the award. To simplify the graphical exposition, it is assumed here that Y is fixed at Y_0; the full optimization (contained in the Appendix) confirms that this is an adequate approximation for small values of σ_θ^2 and/or σ_ζ^2. Researchers respond by producing m_3 or m_4 units of research. The expected differential net social gain from choosing the prize option, rather than contracting for m_0 research units, the allocation which is optimal in the nonstochastic case in which $\hat{\theta}$ is fixed at zero, is represented by $\frac{1}{2}[\mathit{Area\ VEF}-\mathit{Area\ AVD}+\mathit{Area\ FHG}-\mathit{Area\ HJK}]$. This expression could well be negative, even though this option avoids

[9]In accordance with the advice of a referee, the discussion below concentrates on a heuristic graphical exposition; a mathematical derivation of the expected net gain from using a patent, relative to contracting for m_0 units of research, is relegated to the Appendix. The expected gain from a prize is a special case of the gain from a patent, with L and σ_ζ^2 fixed at zero.

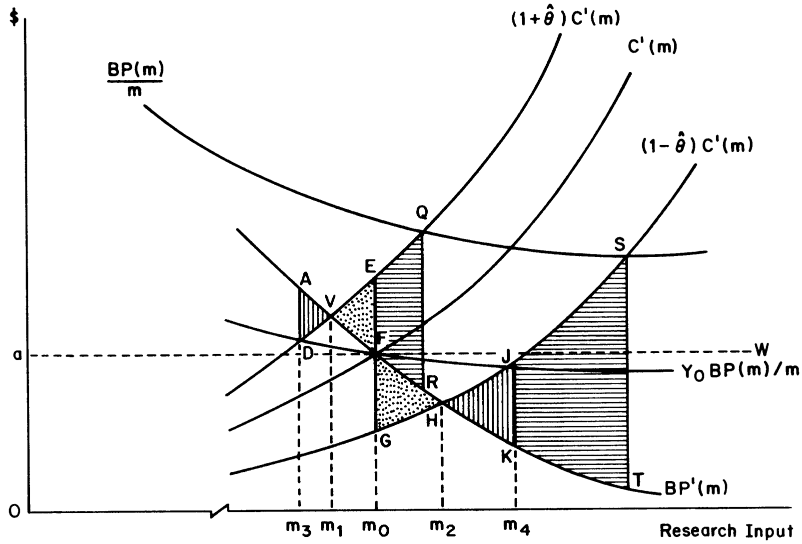

FIGURE 1. THE SOCIAL VALUE OF PRIZES WHEN RESEARCHERS HAVE EXCLUSIVE COST INFORMATION

the alternative welfare loss, represented by $\frac{1}{2}$[*Area EQRF* + *Area JSTK*], which would be incurred if a prize were offered equal to B, the social value of success.

Figure 1 can be used to examine an alternative to the prize option, namely a commitment by the planner to purchase units of research activity at a fixed price, announced at time 0, represented by the horizontal dashed line aW. (This option is equivalent to the "price" option for pollution control presented by Weitzman, 1974.) It is easy to show that the prize option is superior to this alternative; its advantage is that it offers an incentive with a desirable positive correlation with the equilibrium marginal productivity of research.

So far I have illustrated the gains from choosing a prize rather than direct contracting, for just one example. However, as shown in the Appendix, it is possible to derive more general results from this analysis, expressed in terms of elasticities, for cases in which the stochastic shifters ζ and θ have sufficiently small variance. For a prize, the derivation can be illustrated as follows.

An alternative measure of the expected net differential gain in Figure 1 is [*Area DEF* − *Area AFD* + *Area FJG* − *Area FJK*]. We can approximate these areas with the areas of triangles bounded by one straight line through point E, two through F, and one through G, with slopes equal to $C''(m_0)$, $BP''(m_0)$, $C''(m_0)$, $\partial[Y_0 BP(m_0)/m_0]/\partial m$, and $C''(m_0)$, respectively.

Areas DEF and FHG have a vertical side of length $a\hat{\theta}$. Triangles approximating areas AFD and FJK have a vertical side of length $(\partial[Y_0 BP(m_0)/m_0 - BP'(m_0)]/\partial m)\Delta m$, where Δm approximates the distances $(m_0 - m_3)$, and $(m_4 - m_0)$,

$$\Delta m = a\hat{\theta} \Big/ \frac{\partial}{\partial m}\left[C'(m_0) - \frac{Y_0 BP(m_0)}{m_0}\right].$$

Thus if $\hat{\theta}$ is sufficiently small, the expected net differential social gain from a prize relative to contracting for m_0 units of research, $G(prize)$, can be approximated as the expected difference between two triangles:

$$(5)\quad G(prize) = \frac{1}{2}E\left\{a\hat{\theta}\Delta m - \frac{\partial}{\partial m}\left[Y_0\frac{BP(m_0)}{m_0} - B\cdot P'(m_0)\right](\Delta m)^2\right\}.$$

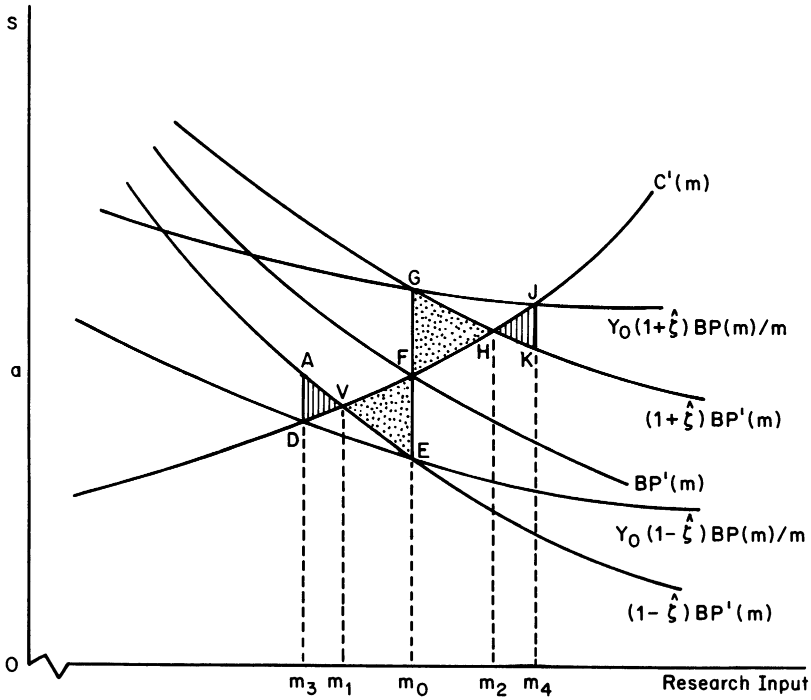

FIGURE 2. THE SOCIAL VALUE OF PATENTS WHEN RESEARCHERS HAVE EXCLUSIVE INFORMATION ABOUT THE BENEFITS OF SUCCESS

Substitute the expression for Δm above, multiply above and below by m_0/a, and use equation (4) to obtain

$$(6)\quad G(\textit{prize}) = \frac{a m_0 \sigma_\theta^2}{2[1/\eta - h(AP)]} \times \left\{1 + \frac{h(MP) - h(AP)}{1/\eta - h(AP)}\right\},$$

where $\eta > 0$ is the elasticity of supply of m at m_0, $\eta(AP) = (m_0 P'(m_0)/m_0 - 1) < 0$ is the elasticity of the average probability of success, $P(m_0)/m_0$, and $h(MP) = m_0 P''(m_0)/P'(m_0) < 0$ is the elasticity of the marginal probability of success.

If the patent welfare cost L equals zero, the gain from choosing a prize in the case illustrated in Figure 1 also equals the gain from choosing a patent. More generally, the difference between the social value of prizes shown in Figure 1 and the social evaluation of patents is the difference between two components. The first is the expected social value arising from the fact that patents, unlike prizes, give researchers an incentive to use exclusive information regarding benefits B, and the second is the patent deadweight loss L. The value of the first component is explored in Figure 2, in which L and $\hat{\theta}$ are assumed to be fixed at zero. Benefits B are subject to the random shifter $\tilde{\zeta}$, which takes either of two values $\pm\hat{\zeta}$, each with a probability of one-half. Either m_3 or m_4 units of research are produced. The expected net social benefit of patents is represented by $\frac{1}{2}[\textit{Area VEF} - \textit{Area AVD} + \textit{Area GHF} - \textit{Area HJK}]$, which is not necessarily positive. An alternative expression for this area is $\frac{1}{2}\{\textit{Area DEF} - \textit{Area DAE} + \textit{Area FJG} - \textit{Area GJK}\}$. These areas can be approximated by triangles bounded by two straight lines through E, one through F, and two through G, with slopes $\partial[Y_0 BP(m_0)/m_0]/\partial m$, $BP''(m_0)$, $BP''(m_0)$, $\partial[Y_0 BP(m_0)/m_0]/\partial m$, and $BP''(m_0)$, respectively.

Areas DEF and FJG in Figure 2 have a vertical side of length $a\hat{\zeta}$. Triangles approximating areas DAE and GJK have a vertical side of length $(\partial[Y_0BP(m_0)/m_0 - BP'(m_0)]/\partial m)/\Delta m$, where Δm approximates the distance $(m_0 - m_3)$, and $(m_4 - m_0)$:

$$\Delta m = a\hat{\xi} \Big/ \frac{\partial}{\partial m}\left[C'(m_0) - Y_0\frac{BP(m_0)}{m_0}\right].$$

Thus if $\hat{\xi}$ is sufficiently small, and independent of θ, and the patent deadweight loss $L = 0$, the expected differential social gain from choosing a patent rather than a prize can be approximated as the expected difference between two triangles, that is,

$$G(\textit{patent})|_{L=0} - G(\textit{prize}) = \frac{1}{2}E\left\{a\hat{\xi}\Delta m - \frac{\partial}{\partial m}\left[Y_0\frac{BP(m_0)}{m_0} - BP'(m_0)\right]\cdot(\Delta m)^2\right\}. \tag{7}$$

Substitution for Δm in (7), use of (4), and multiplication above and below by m_0/a yields the following approximation for the expected net gain illustrated in Figure 2:

$$G(\textit{patent})|_{L=0} - G(\textit{prize}) = \frac{am_0\sigma_\xi^2}{2[1/\eta - h(AP)]} \times \left\{1 + \frac{h(MP) - h(AP)}{1/\eta - h(AP)}\right\}. \tag{8}$$

The expected deadweight loss from a patent scheme, ignored in Figure 2, is approximately,

$$Y_0LP(m_0) = m_0\frac{P'(m_0)}{P(m_0)}\cdot P(m_0)\cdot L = am_0\frac{L}{B},$$

since $BP'(m_0) = a$. Subtraction of this from the sum of equations (8) and (6) reproduces the approximation derived in the Appendix for the expected net social gain $G(\textit{patent})$ from using a patent, relative to opting for m_0 units of contractual research, when Y is optimally adjusted by the administrator, given his information,

$$G(\textit{patent}) = a\cdot m_0\left\{\frac{(\sigma_\xi^2 + \sigma_\theta^2)}{2[1/\eta - h(AP)]} \times\left[1 + \frac{h(MP) - h(AP)}{1/\eta - h(AP)}\right] - \frac{L}{B}\right\}. \tag{9}$$

This approximation, which eliminates terms of smaller order than σ_ξ^2 and σ_θ^2, is appropriate not only for the two-point probability density functions in the examples illustrated in Figures 1 and 2, but also for any independent compact distributions of $\hat{\xi}$ and $\hat{\theta}$ with mean of zero, provided that the variances are sufficiently small. Under the same conditions, equation (6), which is a special case of (9), is an appropriate approximation to the expected social gain from using a prize incentive rather than directly contracting for m_0 units of research.

In the model of the previous section, patent and prize incentives, optimally adjusted *ex ante*, were unaffected by the common pool problem. But Figures 1 and 2, and expressions (6) and (9), show that this is not true when there is an informational asymmetry of the type which one might suppose would justify the more decentralized invention incentives. The common pool problem is then a real drawback of patents and prizes even if the limitation on the award is optimized *ex ante*. To show the extent of the efficiency losses caused by the common pool problem, let us now consider the expected differential net gains from the yardstick marginal patents and prizes discussed in Section I, which are unaffected by the common pool problem even if information is asymmetrically distributed. They thus are consistent with standard analyses of models with no informational asymmetry, in which the common pool problem (as well as any reason for choosing decentralized incentives) does not appear.

[3] *Marginal Patent*. Under the marginal patent, the administrator is again assumed to have the ability to optimally reduce expected

patent value to a fraction, denoted by Z, of its unrestricted level. In this case, the sole rationale for such a "patent life reduction" would be to reduce optimally the excess burden L, as in Nordhaus, who ignores the common pool problem. In this model, the equivalent of patent life limitation is to award the patent with conditional probability Z, given exactly one research project is successful. When L equals zero, Z obviously equals unity.

[4] *Marginal Prize*. Like the marginal patent, this prize is awarded only in the case where just one research effect is successful in making the particular discovery which is the objective of the incentive.

In the cases illustrated in Figures 1 and 2, the expected social gain from choosing a marginal patent relative to contracting for m_0 units of research is represented by $\frac{1}{2}$[*Area VEF* + *Area FHG*], which is nonnegative. The expected advantage of the marginal patent over the more feasible patent considered above, entirely attributable to the interaction of the common pool problem with the informational asymmetry, is represented by $\frac{1}{2}$[*Area AVD* + *Area HJK*] for both the alternate cases illustrated in Figures 1 and 2.

In either figure, the ratio of the triangular approximation for *Area VEF* to the approximations for *Area DEF* derived above is equal to the approximation to $m_0 - m_1$, divided by the approximation to $m_0 - m_3$ made above. This ratio is $[1/\eta - h(AP)]/[1/\eta - h(MP)]$. It is easy to infer from equation (9) and the discussion of its components[10] that

$$(10)\quad G(\textit{marginal patent}) = \frac{am_0\left(\sigma_\zeta^2 + \sigma_\xi^2\right)}{2(1/\eta - h(MP))} - am_0\frac{L}{B};$$

$$(11)\quad G(\textit{marginal prize}) = \frac{am_0\sigma_\theta^2}{2(1/\eta - h(MP))}.$$

[10]A derivation for $G(\textit{marginal patent})$ in equation (11), similar to derivation of $G(\textit{patent})$ in the Appendix, is available from the author.

Subtraction of equation (9) from (10) yields:

$$(12)\quad G(\textit{marginal patent}) - G(\textit{patent}) = \frac{am_0\left(\sigma_\zeta^2 + \sigma_\xi^2\right)[h(AP) - h(MP)]^2}{2[1/\eta - h(MP)][1/\eta - h(AP)]^2}.$$

Similarly

$$(13)\quad G(\textit{marginal prize}) - G(\textit{prize}) = \frac{am_0\sigma_\theta^2[h(AP) - h(MP)]^2}{2[1/\eta - h(MP)][1/\eta - h(AP)]^2}.$$

Given $1/\eta > 0 > h(MP)$, equations (12) and (13) are nonnegative.

The comparisons with the less practical marginal options have shown why the more feasible patents and prizes initially considered cannot motivate researchers holding relevant information unavailable to the public sector to allocate the expected social welfare-maximizing amount of resources to research. *This problem exists even if the researchers have all the available information and know all the relationships in the model, and the administrator optimally sets the patent life or prize value ex ante, given his information set.*

IV. An Illustrative Example

To show how the parameters of the model interact to determine the appropriate choice of incentive when σ_θ^2 and σ_ζ^2 are sufficiently small, to show the cost of choosing a less effective incentive, and to allow checks on the validity of the approximations in Section III when these variances are finite, the following simple example is offered. Because marginal and average probabilities of success are relevant, it is necessary to choose a plausible functional form, rather than just a local approximation, for $P(m)$. In the model of Section II, there is a set of potential research avenues or "projects," each requiring one unit of research input and one period for completion, and each having the same probability of success in finding the single discrete invention goal. If it is further assumed that researchers can recognize which avenues are being pursued by competitors, have sufficient alternative avenues, and can avoid du-

TABLE 1—SIGNIFICANCE OF THE COMMON POOL PROBLEM: THE NET EXPECTED BENEFITS OF THE PRIZE OPTION RELATIVE TO THE YARDSTICK MARGINAL PRIZE[a]

Probability that Success is Achieved[b]	Elasticity of Supply of Research					
	.1	.5	1	2	10	∞
.1	1.00	1.00	1.00	0.99	0.87	c,d
.3	1.00	0.99	0.97	0.92	0.50	c
.6	1.00	0.95	0.86	0.65	c	c
.9	0.98	0.68	0.19	c	c	c

[a]Figures in the body of the table represent $G(prize)/G(marginal\ prize)$ from equations (6) and (11), assuming $\sigma_\theta^2 > 0$.
[b]$P(m) = 1-(1-\pi)^m$, evaluated at the nonstochastic optimum m_0, with $\pi = 0.01$; $P(m)$ is the probability that at least one of m research projects succeeds.
[c]The decentralized options are dominated by a contract for m_0 units of research.
[d]Accurate only for very small values of σ_θ^2.

plication of the research efforts of each other, the total and marginal probabilities of success (i.e., of at least one successful project) are

$$(14)\qquad P(m) = 1-(1-\pi)^m;$$

$$(15)\qquad P'(m) = -(1-\pi)^m \log(1-\pi),$$

where π is the probability of success of any one project, assumed to be small. Then the significance of the common pool problem for sufficiently small σ_ξ^2, σ_θ^2 is indicated by the ratio $G(prize)/G(marginal\ prize)$, reported in Table 1. The results are also bounds on the performance of the patent option relative to the yardstick marginal patent.

Note that toward the bottom right-hand corner of Table 1 the (optimized) prize option is seriously inferior, and for a considerable range of values the contractual allocation of m_0 by the administrator would be preferred, even though it wastes researchers' knowledge of θ. The alternatives are nearly equivalent when the elasticity of supply and the equilibrium probability of success (and the level of research activity) are low. But as supply elasticity falls, the net benefits at stake also decline; incentives are important only if they elicit a nonnegligible response.[11]

If the yardstick marginal options are infeasible, what is the best incentive mechanism from the choice set which includes patents and prizes and direct government contracts? We know, from Table 1, that contracts can dominate prizes. The choice between prizes and patents depends on the tradeoff between the advantage of the latter mechanism, that it uses researchers' superior knowledge of the benefits of success, against its drawback, the deadweight loss associated with the patent.

The patent dominates the prize option if and only if

$$(16)\qquad L/B\sigma_\xi^2 < \frac{1}{2[1/\eta - h(AP)]} \times \left[1 + \frac{h(MP)-h(AP)}{1/\eta - h(AP)}\right].$$

Critical values of $L/B\sigma_\xi^2$ calculated using

[11]Given the number of approximations made in deriving them, the reader may wonder whether these results are accurate for finite σ_θ^2. Consider then the case where the multiplicative shifter θ can take two values, ± 0.2, each with probability 0.5, so that $\sigma_\theta^2 = 0.04$. Further, assume an industry marginal cost function

$$C = (1+\theta)[Am^{(1/\eta+1)}/(1/\eta+1)], A > 0, \eta > 0,$$

where η is the (constant) supply elasticity. Then using equation (14) and searching for optimal reduction of the prize from B, one can confirm that the figures in the first five columns of the table are accurate within 0.01. The least accurate entry is in the upper right-hand corner; 0.87 should be 0.88 in this example. Paradoxically, linear supply curves (for which the approximation (3) is exact) yield greater inaccuracies in this example, for sufficiently low $P(m)$ and high η. At $P(m) = 0.1$, $\eta = 10$, research ceases when $\theta = +0.2$. Therefore Y can be optimally adjusted for $\theta = -0.2$, rendering conventional prizes as efficient as marginal prizes, and superior to contracts. This problem diminishes as more points are added to the distribution of θ, and as σ_θ^2 decreases.

TABLE 2—THE CHOICE BETWEEN PATENTS AND PRIZES: CRITICAL VALUES[a] OF $L/B\sigma_{\zeta}^2$

Probability that Success is Achieved[b]	Elasticity of Supply of Research .1	.5	1	2	10	∞
.1	0.05	0.24	0.45	0.82	2.11	c,d
.3	0.05	0.21	0.36	0.56	0.55	c
.6	0.05	0.16	0.22	0.23	c	c
.9	0.04	0.08	0.03	c	c	c

[a]When $L/B\sigma_{\zeta}^2$ is greater (less) than the number in the text, prizes (patents) are the optimal choice.
[b]$P(m)=1-(1-\pi)^m$, evaluated at the nonstochastic optimum m_0, with $\pi=0.01$.
[c]Patents and prizes are dominated by a contract for m_0 units of research.
[d]Accurate only for very small values of σ_{θ}^2, σ_{θ}^2.

(16) are presented in Table 2. Patents are most likely to be preferable to prizes when the overall success probability is low, and the elasticity of supply of research is moderately high. When probability of success and/or research supply elasticity is high, contracts can dominate patents.[12]

To help the reader judge whether the entries in Table 2 are likely to cover plausible magnitudes of $L/B\sigma_{\zeta}^2$, I offer one specific example. A standard error of 0.25 on the multiplicative shifter ζ would seem to bestow on researchers an ample advantage in their knowledge of the value of the invention.[13] Nordhaus (p. 81) estimates a typical cost reduction from a process invention at around 5 percent, and chooses 1.0 as an average value of the elasticity of product demand. These values imply that L/B (the ratio of the welfare cost of the patent to the sum of the patent revenue and the welfare cost, discussed in fn. 7), is approximately 0.025.

These choices imply that $L/B\sigma_{\zeta}^2$ equals 0.4, which lies in the intermediate range of the values in Table 2. Note that a higher value of σ_{ζ}^2, or a lower value of L/B, would not prevent domination by contracts in the lower right-hand corner of the table. Using the probability of success function (13), the choice between patents, prizes, and contracts, for $L/B\sigma_{\zeta}^2=0.4$, and σ_{θ}^2 positive and small, is illustrated in Figure 3. As indicated, patents are optimal only in the shaded area toward the bottom of the figure. The argument of Machlup and Plant that an inelastic supply of inventors implies that a patent system is undesirable is supported by this analysis. Patents are most attractive at supply elasticities between 2.0 and 4.0, and especially for long shot research areas with low aggregate probability of success; patents are never the best choice if the aggregate probability of success is over 0.45. Prizes are best for research areas with intermediate success probabilities, and for all areas where supply is inelastic, as is likely if there are essential research-specific inputs, unless the overall research venture is almost certain to succeed.

When supply is elastic, contracts are the best choice for the considerable range of values covered by the upper shaded area, and they dominate increasingly as the overall success prospects become more certain or as supply elasticity increases. Thus contracts are best when the research process is most like activities routinely undertaken in other sectors of the economy, where sectoral input supplies are elastic and output is fairly predictable. Paradoxically, the more decentralized incentives are less attractive when they

[12]An accuracy check using a constant elasticity supply function, and with $\zeta=\pm 0.2$, each with probability 0.5, shows the figures in the first five columns accurate within 10 percent, that is, about as accurate as Table 1, given the difference in denominator. Once again, the caveat for linear supply with high η and low $P(m)$ applies.

[13]Note that the coefficient of variation of B could be very much greater than σ_{ζ} if, as one might expect, there are other random shifters, independent of ζ, unobserved by all parties. Only the *relative* difference in information is measure by ζ.

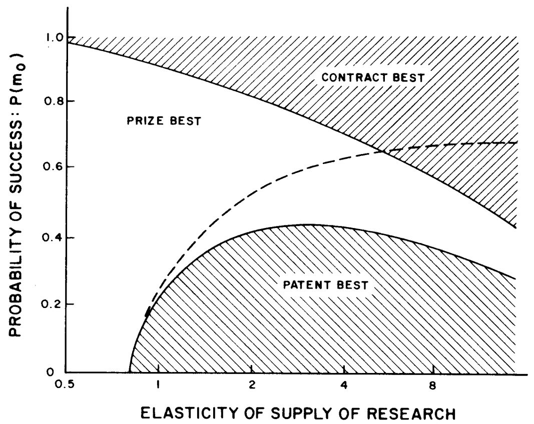

FIGURE 3. THE OPTIMAL CHOICE OF INVENTION INCENTIVE ($L/B\sigma_\zeta^2 = 0.4$)

elicit a highly sensitive economic response! This result is due to the common pool problem; if it did not exist, or if marginal patents and prizes were feasible, patents would be the best choice in Figure 3 below the dashed line, and prizes would be best anywhere above that line.

V. Generalization and Qualifications of the Results

At this point it is appropriate to consider how the choices between incentives change as the assumptions adopted above are modified. First consider the informational assumptions. It can be shown that the relative merit of each incentive does not change if the administrator has exclusive information about costs or returns which is not directly communicated to researchers. By optimally modifying the value of the prize or patent, the administrator can make researchers behave as if they share his knowledge of these random elements.[14] It is only exclusively *private* information which affects the choice between incentives. Furthermore if, instead of shifting $C(m)$, the stochastic term θ shifted $P(m)$ in an analogous fashion, the results derived above would be unchanged. Exclusively private information about the probability function, like exclusively private information about the cost function, increases the value of patents and prizes equally.

The special advantage of patents arises only from *ex ante* researcher information relating to the value of the invention. If, on the other hand, the administrator cannot *ex post* costlessly monitor the quality-adjusted effort of researchers (successful or not), the two more decentralized options should have a further advantage over contracts by allowing self-selection and self-supervision of researchers. In fact, a major real world disadvantage of contracts, ignored here, is the X-inefficiency which can be attributed to the lack of incentives for cost control by researchers in situations where external monitoring is difficult. (However, Yu, pp. 220–21, argues that enforceability problems of contracts have been exaggerated.) An extension of the current model could include differential monitoring costs for each incentive, and model X-inefficiency as a shift in the supply of research. An even more serious problem of moral hazard could arise under patents or prizes, if the patent life or amount of the award were modified *ex post* as a function of input costs, since cost guidelines cannot be as rigidly stipulated as under a contract if the greater private initiative under these options is to be given full rein. There is an additional moral hazard problem from the viewpoint of researchers, in that the government may understate its *ex post* evaluation if it wishes to minimize expenditures, and is not greatly concerned with the effects of such action on the reputation of future governments. For these reasons, and because, as Kitch has emphasized, even the current fixed-life patent system incurs a great burden of litigation, I have not considered awards whose terms are adjusted *ex post*. In any case, where such awards are feasible, patents are inferior to direct grants or prizes because of the presumed higher excess burden of patents. This may explain why we do not observe patents with lives determined *ex post*, whereas, as one referee has noted, most historical examples of prizes were determined *ex post*, though two very important inventions of the modern era, the

[14]An analysis of these cases is available from the author.

chronometer and canning, were awarded predetermined prizes for prespecified achievements, like the prizes considered here.[15]

Finally, the extension to an intertemporal framework of the type pioneered by Barzel appears to be quite feasible for the analysis of patent and prize options presented here. However, as mentioned above, anticipation of severe practical difficulties with implementation of the marginal patent and marginal prize options in an intertemporal context is reflected in the fact that they are treated here as convenient yardsticks rather than as realistic alternatives.

VI. Conclusions

In the model presented here, the peculiar informational advantage of patents over prizes is that they incorporate in the allocative process researchers' information about the value of successful inventions, if such information is unavailable to the public research administrator. Both patents and prizes utilize private information about research costs, or the probability of success. However, information held exclusively by the public administration has no effect on the choice between all three options, optimally implemented.

The informational advantages of the more decentralized research incentives do not necessarily translate into an allocative advantage. Though the existence of exclusively private information is necessary to render patents and prizes superior to contracts, it is by no means sufficient, because of the common pool problem. Even if the value of patents and prizes can be adjusted by a fraction that is optimal *ex ante*, such reduction does not eliminate the effects of this problem in prizes and patents, as it does in previous analytical models where all agents have identical information. When this problem is recognized, the paradoxical result can emerge that contracts, the centralized alternative, are more likely to be the best choice if researchers are highly responsive to incentives than if they are less responsive! If contracts dominate prizes, then patents are the worst choice of all in the model presented here. If contracts are inferior, the incentive choice, for a patentable invention, rests on the tradeoff between the excess burden of the patent system and its informational advantage over prizes.

More specific conclusions about the choice of incentive depend on the nature of the search process, and just one of many plausible models is analyzed in Section IV. Future studies could investigate alternative search processes and relaxation of the assumptions of a discrete goal of research, fully competitive researcher supply, risk neutrality, and the homogeneity of researcher attributes. As a prelude to such further investigation, this paper suggests that the range of situations in which a practical patent system dominates other feasible alternatives may be narrower than is commonly believed, while the relative advantages of contractual research and of prizes may well have been undervalued.

Appendix: Derivation of the Expected Net Social Gain From Choosing the Patent Option

If the patent instrument with conditional award probability Y is adopted in the model of Section II in which multiplicative stochastic shifters ζ and θ are introduced, the difference in the incentive from equation (3), which refers to the model of Section I, is

$$\left[YB(1+\zeta)(1-L/B)Pm^{-1}-Y_0B\bar{P}m_0^{-1}\right].$$

Denote a first-order approximation to this expression, evaluated at $m=m_0$, $Y=Y_0$, $\zeta=L=0$, by ΔI,

$$\text{(A1)}\quad \Delta I=Y_0\frac{B}{m_0}P'(m_0)(m-m_0)+Y_0\frac{B\bar{P}}{m_0}\left(\zeta-\frac{L}{B}\right)+\frac{B\bar{P}}{m_0}(Y-Y_0).$$

[15]Awards of £10,000, £15,000, and £20,000 were offered by the British Board of Longitude in 1713 for a chronometer which measured longitude to within 60, 40, and 30 minutes, respectively. John Harrison claimed the £20,000 reward in 1762, and full payment was completed by 1773 (*Encyclopaedia Britannica*, 1929, Vol. 11, p. 220). In 1795 a prize of 12,000 francs was offered by Napoleon's Society for the Encouragement of Industry for a method of food preservation useable by the military forces. It was awarded in 1810 to Nicolas Appert for a method of food canning using heat treatment of food in sealed champagne bottles (William Harris and Judith Levey, 1975, p. 127; James Burke, 1978, p. 234).

Research firms' responses, which determine m, are based on their knowledge of the stochastic terms and the announced value of Y. In equilibrium, the number of active firms, each producing one unit of research, is

$$\text{(A2)} \qquad m = m(\zeta, \theta, Y).$$

Relative to the equilibrium of Section I, the equilibrium change in its marginal and average cost $c_1(1, m, \theta)$ perceived by any individual research firm equals the change in industry marginal cost $C_1(m, \theta)$. A first-order approximation to this expression, evaluated at $m = m_0, Y = Y_0, \theta = 0$, is

$$\text{(A3)} \qquad \Delta C_1 = a\theta + b(m - m_0),$$

where $a \equiv C'(m_0)$, $b \equiv C''(m_0)$.

Definition of m_0 as the equilibrium level of m when L, ζ and θ are all fixed at zero implies that

$$\text{(A4)} \qquad a \equiv B \cdot P'(m_0).$$

Equating (A1) to (A3) and using (A2),

$$\text{(A5)} \quad m(\zeta, \theta, Y) - m_0 = \left[a\theta - \frac{1}{m_0}(Y_0 B\bar{P} + Y_0 \bar{P} L - B\bar{P}(Y - Y_0)) \right] \Big/ W,$$

where

$$\text{(A6)} \qquad W = Y_0 \frac{a}{m_0} - Y_0 \frac{B\bar{P}}{m_0^2} - b.$$

The administrator sets the fraction Y so as to maximize expected net social benefits, given that the patent system is chosen. Throughout the analysis that follows the second-order conditions of maximization are assumed to be satisfied, and corner solutions are ruled out. Under these conditions, the first-order condition for maximization of net social benefits expected by the administrator is

$$\text{(A7)} \quad E_G\{(1+\zeta)(1 - L/B) \times [a + Bf(m(\zeta, \theta, Y) - m_0)] - (1+\theta)[a - b(m(\zeta, \theta, Y) - m_0)]\} = 0$$

where E_G denotes the expectation of the government administrator conditional on his information in time 0, and $f \equiv P''(m_0)$. Substituting (A5) in (A7) using (A4),

$$\text{(A8)} \quad \frac{1}{m_0} \cdot B\bar{P}(Y - Y_0) = -\frac{Wa\frac{L}{B} - \sigma_\zeta^2 Y_0 \bar{P} B^2 f m_0^{-1} - ab\sigma_\theta^2}{(B - L)f - b} + \frac{Y_0 \bar{P} L}{m_0}.$$

Substituting (A8) in (A5) and using equation (4), and discarding terms of smaller order than σ_ζ^2 and σ_0^2,

$$\text{(A9)} \quad m(\zeta, \theta, Y) - m_0 = a \cdot W^{-1}\left[\zeta - \theta + \frac{\sigma_\zeta^2 Bf + b\sigma_\theta^2}{(B - L)f - b}\right] + \frac{a\frac{L}{B}}{(B - L)f - b}.$$

The expected cost of the patent distortion is $E_G[P \cdot Y \cdot L]$. The net differential social gain expected by the administrator from choosing the limited patent instrument in time 0 rather than directly contracting for research, $G(patent)$ is

$$\text{(A10)} \quad G(patent) = E_G\Big[(1+\zeta)\Big[a(m(\zeta, \theta, Y) - m_0) + \frac{Bf}{2}(m(\zeta, \theta, Y) - m_0)^2\Big] - (1+\theta)\Big[a(m(\zeta, \theta, Y) - m_0) - \frac{b}{2}(m(\zeta, \theta, Y) - m_0)^2\Big] - Y \cdot L \cdot P\Big].$$

Substituting (A9) and (A6) in (A10) and

discarding terms of smaller order than σ_ζ^2 and σ_θ^2,

$$\text{(A11)}\quad G(patent) = am_0 \left\{ \left[\frac{\sigma_\zeta^2 + \sigma_\theta^2}{\left(\frac{1}{\eta} - h(AP)\right)} \right] \times \left[1 - \frac{\frac{1}{\eta} - h(MP)}{2\left(\frac{1}{\eta} - h(AP)\right)} \right] - \frac{L}{B} \right\}$$

where $\eta \equiv a/bm_0 \geqslant 0$ is the elasticity of supply of research activity at m_0, and $h(MP) = bfm_0/a$ and $h(AP) = am_0/\bar{P} - 1$ are the elasticities of the marginal and average probabilities of success, respectively. Rearranged, (A11) becomes equation (9) in the text.

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